ENTRANCE EXAMINATION PROGRAM

**FOR PHYSTECH SCHOOL OF APPLIED MATHEMATICS AND INFORMATICS
MATHEMATICS AND MECHANICS
COMPETITIVE GROUP**

FOR APPLICANTS ENTERING PHD PROGRAMS

At the entrance examination applicants will be asked questions on their final qualifying work and questions from the section corresponding to specialty of their future research activity.

Questions on the final qualifying work (master or specialist's degree):

1. Main provisions.
2. Novelty.
3. Relevance.

Questions for the following specialties:
1.1.6. Computational Mathematics
1.1.7. Theoretical Mechanics

Mathematical analysis

1. Limits of a sequence. Cauchy criterion. Existence of a limit for a monotonically increasing sequence bounded above. The Bolzano-Weierstrass theorem on the existence of a convergent subsequence of a bounded sequence.
2. Number series. Cauchy criterion. Convergence tests (comparison tests, d'Alembert's ratio test, Leibniz's theorem, Dirichlet's test).
3. Limit of a function. Continuous functions. Properties of continuous functions on an interval (Weierstrass's theorems on the boundedness and attainability of the upper and lower bounds. Cauchy's intermediate value theorem). Generalizations of multidimensional case. Existence of one-sided limits for monotone functions. Theorem on the continuity of an inverse function and a continuous monotone one. Uniform continuity.
4. Differentiable functions of one and several variables. Derivatives and differential. Taylor's theorem for functions (one and several variables). Taylor series. Elementary functions. Implicit function theorem (without proof).
5. Calculation of limits using the Taylor's theorem. Sufficient conditions for the monotonicity of a differentiable function. Convex functions. Sufficient conditions for the convexity of a function twice differentiable on an interval. Asymptotes.
6. Extrema of functions of one and several variables. Necessary conditions for an extremum. Sufficient extremum conditions for differentiable functions.
7. Riemann integral. Necessary and sufficient conditions for Riemann integrability of a function. The mean value theorem. Newton- Leibniz formula. The formula for integration by parts. Improper integrals. Convergence tests of improper integrals. Dirichlet's test.
8. The concept of a multiple integral according to Riemann. Reduction of a multiple integral to an iterated one. Change of variables in multiple integrals.
9. The concept of a smooth curve, a smooth surface, their parametric assignment. Determining the length of a curve, the area of a piece of surface. Curvilinear integrals of the first and second kind. Surface integrals of the first and second kind.

10. Green's theorem in the plane. Gauss-Ostrogradsky theorem. Stokes theorem. Differential operations. Gradient, divergence, curl (vortex). Curvilinear integrals independent of the path integration. Potential vector fields. Total differential, necessary conditions, sufficient conditions.
11. Functional sequences and series. Uniform convergence. Cauchy’s convergence test. Weierstrass test for uniform convergence of continuous functions. Theorems on term-by-term integration and differentiation of functional series.
12. Integrals depending on a parameter. Uniform convergence of an improper integral depending on a parameter. Theorems on the continuity and differentiability of integrals depending on a parameter.
13. Fourier series in the trigonometric system. Convergence of Fourier series for piecewise smooth functions. Decreasing order of the Fourier coefficients for an l-times continuously differentiable function. Uniform convergence of the Fourier series for a continuously differentiable function. Uniform approximation of continuous functions on a segment by trigonometric polynomials.
14. Minimal property of Fourier coefficients. Bessel's inequality and Parseval's identity. The concept of Hilbert space and abstract Fourier series in terms of a complete orthonormal system. Theorem on the convergence and mean of Fourier series in the trigonometric system for a function that is integrable.
15. Fourier transforms. Treatment formula. Fourier transform of the derivative and derivative of the Fourier transform.

Linear algebra

1.The concept of a linear space. Definition of linear dependence and independence of vectors. The dimension of a linear space. Basis, vector coordinates, coordinate transformation equations when moving from one basis to another.

2.Matrices and actions on them. Determinant of a square matrix. Matrix rank. The equivalence of its two definitions in terms of the linear independence of the rows (or columns) of the matrix and in terms of the inequality of minors to zero.

3.Systems of linear algebraic equations. Solution of a homogeneous system. Solution of an inhomogeneous system of linear equations. Kronecker-Capelli compatibility criterion.

4.Linear transformations in n-dimensional space. Linear transformation matrix and its meaning. Changing the linear transformation matrix when changing the basis. The range of the linear transformation and its matrix. Product of linear transformations.

5.Eigenvectors and eigenvalues of linear transformations. Characteristic polynomial. Linear independence of eigenvectors corresponding to different eigenvalues. Linear transformation matrix in the basis of eigenvectors. Jordan basis of linear transformation and Jordan normal form (without proof).

6.Dot product in Euclidean space. Coordinate representation of the dot product. Orthonormal basis. orthogonalization process.

7.The concept of a self-adjoint linear transformation. Properties of its eigenvalues and eigenvectors. Self-adjoint transformation matrix.

8.Orthogonal transformations. Orthogonal transformation matrix. Orthogonal matrices. Transition from one orthogonal basis to another.

9.Bilinear and quadratic forms. Their matrices and formulas for the transition from one basis to another. Carrying out a quadratic form to a canonical form in an orthonormal basis. Law of inertia for quadratic forms. The concept of a positive definite quadratic form. Sylvester's criterion (without proof).

Ordinary differential equations

1.Elementary methods for integrating first-order equations (equations with separating variables, homogeneous equations, linear equations, Bernoulli equations, equations in total differentials).
2.Existence and uniqueness theorems for the solution of the Cauchy problem for one 1st order equation and for a system of n 1st order equations with n unknowns in normal form (without proof). Specificity of the case of linear differential equations.
3.Linear equations of the nth order with constant coefficients. Solution of a homogeneous equation. Solution of an inhomogeneous equation with a special right-hand side in the form of a quasi-polynomial. Euler equation.
4.Solution of a homogeneous first order system with constant coefficients (the case of simple roots).
5.Linear equations of the nth order with variable coefficients. Fundamental system of solutions of a homogeneous equation and its existence. Wronskian. Liouville's formula. Possibility of lowering the order of a homogeneous equation. Solution of a homogeneous equation. Solution of an inhomogeneous equation. Method of variations of arbitrary constants.
6.Systems of linear equations of the first order with variable coefficients. Fundamental system of solutions of a homogeneous system and its existence. Liouville formula. The method of variation of arbitrary constants in finding a particular solution of an inhomogeneous system. The structure of the general solution.
7.The concept of equations that are not resolved with respect to the highest derivative. Special solution.
8.Autonomous systems. Equilibrium position. Phase plane and phase trajectories. Classification of equilibrium positions in the plane. The concept of stability of an equilibrium position according to Lyapunov and asymptotic stability. Theory of stability in linear approximation.
9.The first integrals of an autonomous system. Linear homogeneous equations in partial derivatives of the first order. General view of the solution. Cauchy problem. The concept of characteristics.
Elements of the calculus of variations. 10.The simplest problem of the calculus of variations and its simple generalizations. Constrained variational problem, isoperimetric problem.

Theory of functions of a complex variable

1.Function of one complex variable. Differentiable functions of a complex variable. Cauchy-Riemann equations. The geometric meaning of the modulus and argument of the derivative of a function of a complex variable.
2.Equality to zero of the integral of a differentiable function over a closed contour contracting to a point. Cauchy's integral formula.
3.The concept of a regular function at a point and in a region. Power series. Abel's first theorem. The circle of convergence of a power series. Term by term differentiation and integration of power series. Equivalence of differentiability and regularity of function and domain. Regularity of uniformly convergent series and regular functions.
4.Expansion in a Taylor series of a function differentiable in a neighborhood of a point. Laurent series. Elementary Functions Z^n, e^Z, *Sin Z, Cos Z, Sh Z, Ch Z,* etc.

5.Isolated Singular Points of Single Character. Classification: removable singular point, pole, essential singular point. Characterization of the singular point of a function in terms of the coefficients of the Laurent series.
6.The concept of a residue at an isolated singular point of a single-valued nature. Calculation of contour integrals using residues.
7.Decomposition of meromorphic functions into elementary fractions. Infinite products. Examples of decomposition of some entire functions into infinite products.
8.The uniqueness theorem for a regular function taking given values on a sequence of points, the limit of which is contained in the domain of regularity. Analytic continuation. The concept of a

complete analytic function. Main multivalued elementary functions ______ ln Z. The concept of a Riemann surface
9.Conformal mappings implemented by regular functions. The concept of a univalent mapping. Fractional linear mappings and their properties. Mappings carried out with the help of some elementary functions. Riemann's general theorem on the existence of conformal mappings (without proof). The principle of correspondence of boundaries under conformal mapping.

Equations of Mathematical Physics
1.Linear differential equations in partial derivatives of the 2nd order. Reduction to canonical form at a point. Classification of equations. Elliptic, hyperbolic and parabolic equations.
2.Linear differential equations of the 2nd order in the plane. The concept of characteristics. Reduction to the canonical form in the domain. Cauchy problems. Cauchy-Kovalevskaya theorem (without proof).
3.The concept of a well-posed boundary value problem for a partial differential equation. Examples of some problems (Cauchy problems for the Laplace equation). Statement of classical problems of mathematical physics and their physical meaning (the Cauchy problem and the mixed problem for the string vibration equation, for the heat equation, the Dirichlet and Neumann boundary condition for the Laplace equation).
5.Fredholm integral equations. Integral equations with degenerate kernel. Fredholm's theorem for Fredholm integral equations of the second kind with a continuous kernel (without proof). Generalization to the case of polar nuclei. The method of successive approximations and the Neumann series.
6.The Sturm-Liouville theory. Green's function of a boundary value problem for an ordinary differential equation. Reduction of the Sturm-Liouville theory to an integral equation. Properties of eigenvalues and eigenfunctions of the Sturm-Liouville theory.
7.Cauchy problem for the wave equation. D'Alembert's formula in the case of the string vibration equation. Existence and uniqueness of a solution. The area of dependence of the solution on the initial data.
8.Mixed problems for hyperbolic equations. Fourier method (method of separation of variables). Uniqueness of the solution.
9.The Cauchy problem for the heat equation. The existence and uniqueness theorem. Poisson formula. Fundamental solution and its meaning.
10.Mixed problem for the heat equation. Fourier method (method of separation of variables). Uniqueness of solution, maximum principle.
11.Laplace and Poisson equations. Harmonic functions and their properties. Green's formulas. The mean value theorem for harmonic functions. Principle of maximum and minimum for harmonic functions.
12.The Dirichlet problem for the Laplace equation. Fundamental solution. The concept of Green's function for the Dirichlet problem. Solution of the Dirichlet problem for the Laplace equation in a circle
Fourier method. Existence and uniqueness of a solution to the Dirichlet problem in the general case (without proof).
13.The Neumann problem for the Laplace and Poisson equations. Necessary and sufficient conditions for its solvability. Degree of decision uncertainty.

References

Mathematical analysis

1. Кудрявцев Л.Д. Курс математического анализа, т. 1 и т. 2.
2. Никольский С.М. Курс математического анализа, т. 1 и т. 2.
3. Фихтенгольц Г.М. Курс дифференицального и интегрального исчисления, т. 1, т. 2 и т.3.

4. Смирнов В.И. Курс высшей математики, т. 1 и т. 2.

Linear algebra

1. Беклемишев Д.В. Курс аналитической геометрии и линейной алгебры.
2. Гельфонд И.М. Лекции по линейной алгебре.
3. Курош Л.Г. Курс высшей алгебры.

Ordinary differential equations

1. Федорюк М.В. Обыкновенные дифференциальные уравнения.
2. Степанов В.В. Курс дифференциальных уравнений.
3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений.
4. Понтрягин Л.С. Обыкновенные дифференциальные уравнения.
5. Смирнов В.И. Курс высшей математики, т. 2.

Theory of functions of a complex variable

1. Сидоров Ю.В., Федорюк М.В., Шабунин М.И., Лекции по теории функций комплексного переменного.
2. Лаврентьев М.А., Шабат Б.В. Методы теории функций комплексного переменного.
3. Привалов И.И. Введение в теорию функций комплексного переменного.
4. Маркушевич А.И. Теория аналитических функций т. 1. и т. 2.

Equations of Mathematical Physics

1. Тихонов В.Н., Самарский А.А. Уравнения математической физики.
2. Петровский И.Г. Лекции об уравнениях с частными производными.
3. Владимиров В.С. Уравнения математической физики.
4. Смирнов В.И. Курс высшей математики т. 2 и т. 4.

Computational Mathematics

1.Solution of systems of nonlinear equations. Newton's method. Theorem on the quadratic rate of convergence. Simple iteration methods, convergence analysis. Parameter continuation method.
2.Numerical differentiation. Basic difference approximations of the first and second derivatives. Approximation error, rounding error. The optimal step of numerical differentiation.
3.Numerical integration of the Cauchy problem for ODE systems. Grid method, simplest difference schemes (Explicit and implicit Euler schemes, central differencing scheme). Implementation of difference schemes. Approximation error, criteria for small grid spacing.
4.Methods of Runge-Kutta type, basic construction, implementation algorithm. Problem of convergence method. Stability of Runge-Kutta methods. Convergence theorems under different proposals for the matrix fx(x).
5.ODEs stiff equations. Numerical methods of solution. A - stable, A(α) - stable methods. Asymptotic stability.
6.Boundary value problems for ODE systems. Statement of a general (multipoint) boundary value problem. Linear boundary value problems, their solution of the problem to the fundamental solution system. Reduction of a linear boundary value problem to Cauchy problems.
7.Nonlinear boundary value problems for ODE systems. Shooting method, quasi-linearization method (like Newton's method in functional space).
8.Linear boundary value problems with a large parameter. Computational instability of the simplest reduction to Cauchy problems. Sweeping method. Equation for sweep coefficients. Reductions to stable Cauchy problems.

9.Sweeping in the Sturm-Liouville difference problem. Difference sweeping algorithm, recurrent formula.
10.Mesh method for the heat equation. The simplest difference schemes (explicit, implicit). Approximation of equations, initial and boundary conditions. Explicit schema implementation. Layer count. Implementation of the implicit scheme, the equation on the upper layer, its solution by the sweeping method.
11.Nonlinear equations, their difference approximation and implementation of the corresponding schemes. Schemes with non-linearity on the upper layer, their implementation (Newton's method and sweeping method).
12.Stability of difference schemes as a continuous dependence of the solution on the input information. Spectral stability. Spectrum calculation technique. The practice of applying the spectral stability. The principle of frozen coefficients. Spectral stability and stability according to initial data. Stability of boundary conditions.
13.Two-dimensional heat conduction equation. Explicit and implicit schemes. The problem of solving equations in the upper layer. Method of variable directions. Its implementation, the cost-effectiveness of the method. Spectral stability. Method of variable directions in three-dimensional problems. Schemes with a factorized regularizer. Spectral stability of such schemes. Splitting method, circuits with an excluded intermediate layer.
14.Solutions of the Poisson equations by the grid method. Difference approximation of the Poisson equations. Method of simple iterations, error, discrepancy. Spectrum of the Poisson difference problem, eigenvalues and functions. Spectral analysis of the convergence of simple iterations. Choice of the optimal iterative parameter. Estimation of the number of iterations. Chebyshev acceleration method for simple iterations. Stability analysis. Stable renumbering of iterative parameters.
15.The method of variable directions for solving the Poisson difference equation. Spectral analysis of convergence. Choice of the optimal iterative parameter. Estimation of the number of iterations. Method of alternating directions with a series of parameters.
16.Numerical methods for solving problems of continuum mechanics. The idea of constructing difference schemes. Conservative methods.
17.Methods for searching for extrema of functions
18.Ill-posed tasks. Examples. Qualitative description of the approach to their solution. The role of a priori information. Examples - integral equation of the 1st kind. The inverse problem of heat conduction.
19.The main idea of regulation. A priori information. Mathematical formalism. The role of choosing the norm in the concept of incorrectness. natural norms. Tikhonov correctness set. Theorem on the continuity of the inverse mapping on the image of a compact. Compact as a mathematical equivalent of a priori information. Quasi-solution approach. Theorem on the continuity of the quasi-solution.

Reference

1. В.С. Рябенький. Введение в вычислительную математику. М.: Физматлит, 2009г., 294с.
2. Р.П. Федоренко. Введение в вычислительную физику. Долгопрудный. Издательский дои Интеллект, 2000 г., 503 с.
3. Н.Бахвалов, Н. Жидков, Г. Кобельков. Численные методы. М.-СПб: Физматлит, 2000г.,622с.
4. И.Б. Петров, А.И. Лобанов. Лекции по вычислительной математике. 2006г., 522с.

Computational geometry

1.The concept of a point and a vector. Appropriate data structures.
2.Dot product of vectors.

3.Vector product.
4.Oriented area of a triangle. The area of an arbitrary simple polygon.
5.Clockwise predicate. Intersection test of segments without calculating the intersection point.
6.Distances from a point to a line, from a point to a segment.
7.Finding the point of intersection of two lines. Normal equation of a straight line.
8.Intersection of a circle and a straight line. The intersection of two circles.
9.Convex hull (with complexity O(N log N))
10.Scan line method.

Parallel programming
1.Varieties of parallel architectures. SISD - MIMD
2.Varieties of parallel architectures. Common and shared memory.
3.MPI - definition and basic principles, groups and communicators (MPI_COMM_WORLD, MPI_Comm_rank(),...)
4.MPI_Init() and MPI_Finalize()
5.MPI_Send() and MPI_Recv()
6.Blocking and non-blocking passing, MPI_Isend() and MPI_Irecv()
7.Blocking and non-blocking receive, MPI_Sendrecv() and MPI_Rsend()
8.MPI_Bcast()
9.MPI_Reduce()
10.MPI_Scatter() and MPI_Gather(), MPI_Barrier()
11.Sorting and their parallelization, all2all and all2one.
12.Schemes of interaction of processes in sorting
13.Scheme of interaction of the hypercube type and its advantages.
14.Parallelization of array aggregation, fixed-sampling integration.
15.Amdahl's law (application and limitations)
16.Gustafson-Barsis law (application and limitations)
17.Batcher sorting. Resource allocation scheme.
18.Data decomposition, MPI_Status_ignore
19.Topologies, MPI_Cart_create() and MPI_Cart_coords()
20.Topologies, MPI_Cart_sub(), MPI_Cart_rank()
21.Topologies, MPI_Cart_Cart_get(), MPI_Cartdim_get()
22.MPI_Cart_shift() and vector operations
23.Decompositions of nonuniform grids
24.Wait functions: MPI_Wait(), MPI_Test(), using MPI_Status()
25.Graphics accelerators.
26.CUDA Technology
27.OpenCL language.
28.Programmable logic integrated circuits (FPGA).

References

1. Андреев С.С., Дбар С.А., Лацис А. О., Плоткина Е. А. Некоторые проблемы реализации вычислений на FPGA- ускорителях // Научный сервис в сети Интернет: труды XVIII Всероссийской научной конференции (19-24 сентября 2016 г., г. Новороссийск). — М.: ИПМ им. М. В. Келдыша, 2016. — С. 9-13. — URL: http://keldysh.ru/abrau/2016/32.pdf

Questions for specialty:

1.1.5. Mathematical logic, algebra, number theory and discrete mathematics

1. **Mathematical programming**

Convex sets, convex functions, strongly convex functions, their properties. Lagrange multiplier rule. Kuhn-Tucker theorem, dual problem, its properties. Gradient projection method Newton's method. Coordinate descent method. Method of penalty functions. Method of barrier functions. Dynamic programming method. Linear programming. Simplex method. Dual problems of linear programming.

2. **Operations research, game theory**

Antagonistic games. Matrix games, minimax theorem. Convex-concave antagonistic games. Saddle point existence theorem. Non-cooperative games of n persons. Nash equilibrium. The principle of guaranteed results. minimax tasks. Multicriteria optimization. Pareto optimality. Lexicographic approach.

3. **Optimal control**

Optimal control problems, their classification. Pontryagin's maximum principle. Boundary value problem of the maximum principle. Linear speed problem, its properties (existence of a solution, number of switches). The maximum principle and the calculus of variations.

4. **Discrete optimization**

Integer linear programming (Gomory's method, unimodularity properties of constraint matrix). The branch and bound method (on the example of problems of integer or Boolean linear programming). Time complexity of solving discrete optimization problems. Main difficulty classes (P, NP, NPC). NP-hard problems (knapsack problem, traveling salesman problem).

5. **Theory of functional systems**

Completeness problem. Theorem on the completeness of systems of functions of two-valued logic P2. Automata. Regular events and their representation in automata. Algorithmic unsolvability of the completeness problem for automata. Computable functions. Equivalence of the class of recursive functions and the class of functions computable on Turing machines. Algorithmic unsolvability of the word equivalence problem in associative calculus.

6. **Algebra of logic, combinatorial analysis and graph theory**

Equivalent transformations of two-valued logic formulas P2. Basic combinatorial numbers. Inclusion-exclusion method. Graphs and networks. Estimates for the number of graphs and networks of various types. Euler and Hamiltonian graphs.

7. **Control systems**

The concept of a control system. The main model classes of control systems: disjunctive normal forms, formulas, contact circuits, circuits of functional elements, automata, Turing machines, operator algorithms. The main problems of the theory of control systems.

8. **Disjunctive normal forms**

The problem of minimizing Boolean functions. Disjunctive normal forms (DNF). Posing the problem in geometric form. Local algorithms for constructing DNFs. Construction of a DNF $\sum T$ (dead-end sum) using a local algorithm.

9. Intelligent systems

Declarative representation of knowledge: frames, semantic networks, ontologies. Procedural knowledge representation: production system. Heuristic search techniques: state space, greedy search, A*, ray search. Heuristic search techniques: gradient descent, simulated annealing, genetic algorithms. Knowledge engineering: working with experts, debugging the knowledge base. Machine learning: training and test set, overfitting, Bayesian and optimization methods.

10. Software architecture

Software architecture design: classes and interfaces in object-oriented programming. Software architecture design: Class design: inheritance and aggregation, generic programming. Software Architecture Design: class design patterns. Factory, Singleton, Wrapper and Adapter patterns.

11. Machine learning

Support vector machine. Existence and uniqueness of solutions. Nonlinear generalization. Statistical learning approach, mean and empirical risk. Bayesian optimal solution. Adjustment of parameters of recognizers, cross and sliding control. Retraining. Boosting, AdaBoost algorithm. Linear classifiers with penalty functions, problem statement. Variants of linear classifiers. Probabilistic linear classifiers. Fisher's linear discriminant. logistic regression.

12. Statistical data analysis

Basic concepts of mathematical statistics: statistical hypothesis, test statistics, actual level of significance. Goodness of fit criteria: testing for uniformity, exponentiality, normality. Models and methods for checking the homogeneity of samples. One-factor and two-factor models of analysis of variance. Criteria for ordered alternatives. Chi-squared test. Cluster analysis algorithms: shortest non-closed path, k-means clustering, FOREL algorithm. Hierarchical procedures. Dendrograms. Pearson and Spearman correlation coefficients. Principal component method. Confidence ellipsoid. Private correlation. Linear regression model. Methods for studying regression residuals. Stepwise regression procedure.

13. Formal languages, parsers

Parsing algorithms for CFG-grammars (CYK and Earley algorithms). Eisner algorithm. Deterministic parsing for dependency trees.

14. Ontologies

The concept of ontology engineering, its history and the main definitions associated with it (concept, instance, attribute). Inheritance of concepts and ontologies. Formal representations of ontologies in IT. RDF, RDFS, OWL languages, their main classes. The set-theoretic approach to the construction of ontologies (BORO-method), its main advantages and disadvantages.

15. Main tasks and methods of automatic text processing

Statistical machine translation. Basic translation equation. Translation models: word-by-word, phrasal, syntactic. Language model (smoothness of translation). The main problems of recognition of named entities. Computer morphology in various NLP tasks. Text deduplication methods.

16. Grammar system of natural language

Give examples from languages known to you in which the establishment of different sets of non-tree links leads to different interpretations. What are the main elements of describing non-tree links in an automatic text processing system? What are the main problems in finding a controller for a non-tree connection in the case of the control of reflexives and third person pronouns? Give examples from languages you know.

Give definitions and/or examples for the following terms: *word form, lexeme, morph, morpheme; inflection, word formation*. Give definitions and/or examples for the following terms: *grammatical category, grammeme, grammatical indicator, (*morphological*) paradigm.*
Give definitions and/or examples for the following terms: *semantic valency, syntactic valency, participant* (= participant in the situation*), syntactic actant* (= argument*), circumstant (*adjunct*), semantic role, syntactic function, diathesis.* Give definitions and/or examples: *constituent grammar, dependency grammar, non-projective dependency structure; syntactic* (= structural*) ambiguity*. List as many as you can remember: *parts of speech* (verb, noun, ...); *types of components* (VP, NP, ...); *semantic roles* (agent, ...); *syntactic functions* (subject, direct object, ...).

Reference

1. Рассел, Норвиг. Искусственный интелект: современный подход.
2. Саймон Хайкин. Нейронные сети: полный курс.
3. Хасти, Тибширани, Фридман. Элементы статистического обучения.
4. Бишоп. Распознавание образов и машинное обучение
5. Лагутин. Наглядная математическая статистика
6. М. А. Кронгауз. Семантика.
7. Дж. Лайонз. Язык и лингвистика: Вводный курс.
8. В. А. Плунгян. Общая морфология: Введение в проблематику.
9. Я. Г. Тестелец. Введение в общий синтаксис.
10. А. Я. Шайкевич. Введение в лингвистику.
11. Manning, Schütze, Foundations of Statistical Natural Language Processing
12. Jurafsky, Martin, Speech and Language Processing
13. Koehn, Statistical Machine Translation
14. Яблонский С.В. Введение в дискретную математику. М.: Высш. школа, 2001.
15. Журавлев Ю.И., Флеров Ю.А. Дискретный анализ. Комбинаторика. Алгебра логики.Теория графов: Учеб. пособие. – М.: МФТИ, 1999.
16. Журавлёв Ю.И., Флёров Ю.А., Вялый М.Н. Дискретный анализ, ч. III. Формальныесистемы и алгоритмы: учебное пособие. – М.: ООО Контакт Плюс, 2010.
17. Теория и реализация языков программирования. Учебное пособие. /В.А. Серебряков,М.П. Галочкин, Д.Р. Гончар, М.Г. Фуругян. – М.: МЗ-Пресс, 2003.
18. Васильев Ф.П. Методы оптимизации. М.: Факториал, 2002.
19. Карманов В.Г. Математическое программирование. М.: Наука, 2000.
20. Тихомиров В.М., Фомин С.В., Алексеев В.М. Оптимальное управление. М.: Наука,2003.
21. Краснощеков П.С., Петров А.А. Принципы построения моделей. М.: Фазис, 2002.
22. Морозов В.В. Основы теории игр. М.: Изд-во МГУ, 2002.